Grey Polynomial Model

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Abstract

In this paper, a grey polynomial model is proposed. In the GM(1,1) modeling, it is assumed that the data after 1-AGO (first-order accumulated generating operation) is of exponential-like form. The 1-AGO preprocessed data is then fit by a first-order difference equation. However, in many cases the 1-AGO preprocessed data may not have an exponential-like form which results in a inaccurate modeling and thus poor performance. By the observation, we replace the first-order difference equation in the GM(1,1) modeling with a polynomial which is the 1-AGO preprocessed data fit into. The simulation results show that the proposed grey polynomial model has better performance than the GM(1,1) model in the given example.

1. Introduction

The grey model is proposed in [1] which consists of three parts: 1-AGO, data fitting with first-order difference equation, and 1-IAGO (first-order inverse accumulated generating operation). The preprocessing scheme 1-AGO is to reduce the randomness inherent in data and to convert the data into exponential-like form. Then by the 1-AGO converted data a first-order difference equation is solved where two parameters, developing coefficient and grey input, are found. Through the two parameters, an estimate is obtained which is then post-processed by 1-IAGO to find the final estimate. The most popular grey model is the GM(1,1) model. In the GM(1,1) modeling, as few as four data are required and no statistical assumption is made on data. However, several publications [2-4] have pointed out that the inherent problem in GM(1,1) model is that the 1-AGO preprocessed data may not have an exponential-like form. If this is the case, the modeling of the first-order difference equation is not appropriate since its solution is of exponential form. Consequently, the performance of GM(1,1) model degrades. To relieve the problem, in the proposed approach a polynomial model is used to replace the first-order difference equation in the GM(1,1) modeling. The proposed approach is then called grey polynomial model (GPM) and denote GPM(N,1) as single variable with a polynomial of degree N. By this modification, the 1-AGO preprocessed data may have better fitting and give a hope for better performance. This paper is organized as follows: A brief review of GM(1,1) model is given in Section 2. Next, the proposed GPM(2,1) model is described in Section 3 and justified in Section 4 where a performance comparison with GM(1,1) model is made as well. Finally, conclusion is given in Section 5.

2. Review of GM(1,1) Modeling

In this section, the GM(1,1) modeling is briefly reviewed. For details, one may consult [1]. Given data sequence $\{x(k) \geq 0,\ 1 \leq k \leq K\}$, the GM(1,1) modeling is described as follows:

Step 1. Preprocess $x(k)$ by 1-AGO as

$$x^{(1)}(k) = \sum_{i=1}^{k} x(i),\ \text{for } 1 \leq k \leq K \qquad (1)$$

Step 2. By $x(k)$ and $x^{(1)}(k)$, form a grey difference equation as

$$x^{(1)}(k) - x^{(1)}(k-1) + az^{(1)}(k) = b, \text{for } 2 \leq k \leq K \qquad (2)$$

where $z^{(1)}(k) = 0.5[x^{(1)}(k) + x^{(1)}(k-1)]$. With $x(k) = x^{(1)}(k) - x^{(1)}(k-1)$, (2) can be written as

$$x(k) = -az^{(1)}(k) + b \qquad (3)$$

Step 3. Find parameters a (developing coefficient) and b (grey input) in (3) as

$$\begin{bmatrix} a \\ b \end{bmatrix} = (\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})^{-1}\boldsymbol{B}^{T}\boldsymbol{y} \qquad (4)$$

where

$$\boldsymbol{y} = \begin{bmatrix} x(2) \\ x(3) \\ \vdots \\ x(K) \end{bmatrix} \text{ and } \boldsymbol{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(K) & 1 \end{bmatrix} \qquad (5)$$

Step 4. Obtain the estimate of $x^{(1)}(k)$, $\hat{x}^{(1)}(k)$, as

$$\hat{x}^{(1)}(k) = [x(1) - \frac{b}{a}]e^{-a(k-1)} + \frac{b}{a} \qquad (6)$$

Step 5. Calculate the estimate of $x(k)$, $\hat{x}(k)$, by 1-IAGO as

$$\hat{x}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \qquad (7)$$

Note that it may cause a numerical problem in (6) as parameter a approaches to zero. To avoid the problem, a transformed GM(1,1) model is proposed in [5] whose final estimate is found as

$$\hat{x}(k) = \left[\frac{b - ax(1)}{1 + 0.5a}\right]\left[\frac{1 - 0.5a}{1 + 0.5a}\right]^{k-2} \qquad (8)$$

Both GM(1,1) model and the transformed GM(1,1) model will be compared with the GPM(2,1) model in Section 4.

3. The Proposed Grey Polynomial Model

The proposed GPM(2,1) model modified from GM(1,1) model is given in this section. In the GPM(2,1), the first-order difference equation is replaced by a second-degree polynomial while preprocessing 1-AGO and post-processing 1-IAGO remain unchanged. Since the second-degree polynomial requires only three data, thus $K = 3$ in (1) for GPM(2,1) model. To convert into GPM(2,1) model, Steps 2 to 4 in the GM(1,1) modeling are modified as follows:

Step 2'. Assume $x^{(1)}(k)$ has the form of second-degree polynomial as

$$x^{(1)}(k) = c_2k^2 + c_1k + c_0 \qquad (9)$$

Step 3'. Substitute $1 \le k \le 3$ into (9) as

$$\begin{bmatrix} x^{(1)}(1) \\ x^{(1)}(2) \\ x^{(1)}(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} c_2 \\ c_1 \\ c_0 \end{bmatrix} \qquad (10)$$

Step 4'. Find the estimate of $x^{(1)}(k)$, $\hat{x}^{(1)}(k)$, by substituting the coefficients c_k found in (10) into (9).

Since the preprocessing 1-AGO is able to reduce the randomness, a second-degree polynomial is sufficient to model the 1-AGO converted data. Therefore, only three data are required in the GPM(2,1) modeling. Since the polynomial is more appropriate to fit the data than a first-order difference equation, thus better performance is expected for GPM(2,1) model than for GM(1,1) model. This is justified in the following section.

4. Simulation Result

In this section, an example is provided to verify the proposed GPM(2,1) model and to compare it with GM(1,1) model. The example given in the simulation is the speech signal, b.wav shown in Figure 1, whose 1-AGO preprocessed data is not of exponential form generally. Here, the performances of data-fitting for GPM(2,1) model and GM(1,1) model are compared. In the simulation, the GPM(2,1) model uses the data as it is. The fitting result and fitting error are shown in Figure 2 which indicates the GPM(2,1) model almost has perfect fitting. For GM(1,1) model, the input data is shifted up by 2 to satisfy the condition $x(k) \ge 0$. Correspondingly, the final estimates obtained from GM(1,1) model are shifted down by 2. The fitting result and error of GM(1,1) model are depicted in Figure 3. As seen in Figure 3, GM(1,1) model has a numerical problem in non-speech segments since parameter a approaches zero in the modeling where the fitting error is clipped to ± 1 if it is out of the range. To avoid the numerical problem happened in the GM(1,1) modeling, the transformed GM(1,1) is applied whose fitting result and error are given in Figure 4. The fitting performance is not quite good in the given example. From Figures 2 to 4, as expected the proposed GPM(2,1) model has much better fitting performance than GM(1,1) and the transformed GM(1,1) models. It implies that the polynomial model is much more appropriate than the first-order difference equation in GM(1,1) model and the transformed GM(1,1) model.

5. Conclusion

In this paper, a grey polynomial model (GPM) is proposed to improve the fitting performance of GM(1,1) model. The motivation for GPM model is that the 1-AGO preprocessed data may not have an exponential-like form. If this is the case, the first-order difference equation is not able to fit the data appropriately. To deal with the problem, a polynomial model takes the place of first-order difference equation in the GM(1,1) modeling. A second-degree polynomial is generally sufficient to fit data accurately, since the randomness has been reduced by 1-AGO. A speech signal is given as an example to verify the proposed GPM model whose fitting performance is compared with GM(1,1) model and the transformed GM(1,1) model. As expected, the GPM model outperforms the other two models.

References

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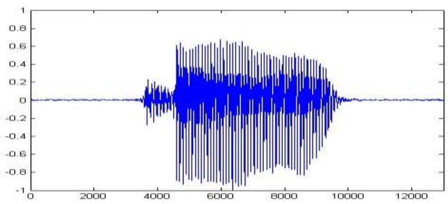

Figure 1. The speech signal b.wav

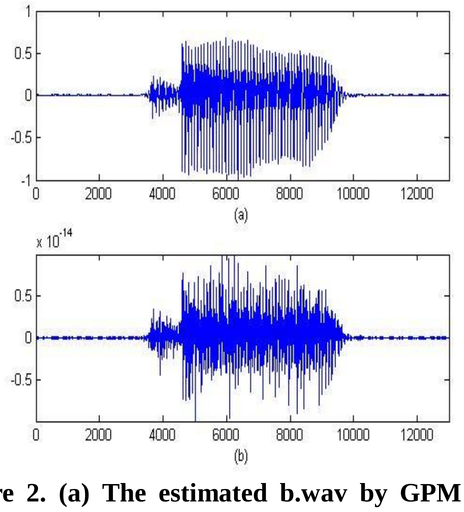

Figure 2. (a) The estimated b.wav by GPM (2,1) model (b) The fitting error in GPM (2,1) model

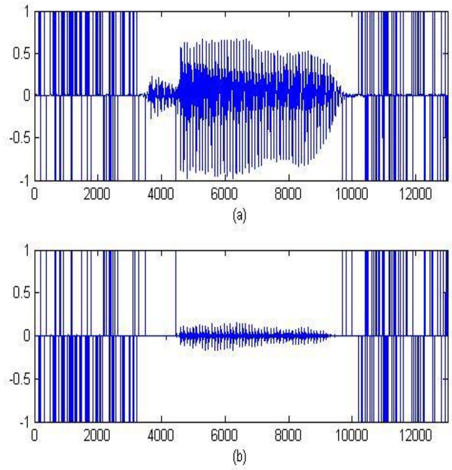

Figure 3. (a) The estimated b.wav by GM (1,1) model (b) The fitting error in GM (1,1) model

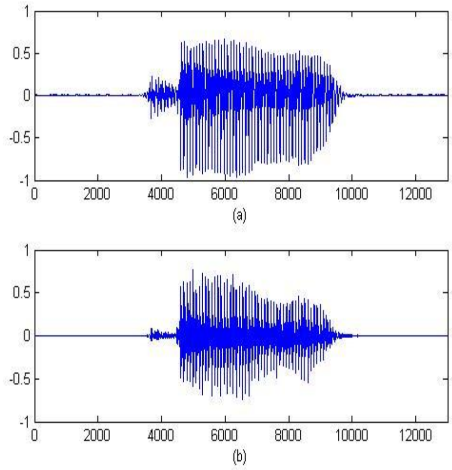

Figure 4. (a) The estimated b.wav by transformed GM (1,1) model (b) The fitting error in transformed GM (1,1) mode